



Define "slide speed" $v_b - v_r$ as function of t such that

$$v_b = v_r + f(t) \Rightarrow \Delta v_b = \Delta v_r + f'(t)\Delta t$$

$$\text{or } v_r = v_b - f(t) \text{ and } \Delta v_r = \Delta v_b - f'(t)\Delta t$$

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$$\text{Total shell travel over time } T = \int_0^T v_b \cdot dt = \int_0^T v_r \cdot dt + L \text{ given slide dist } L$$

Momentum considerations

$$(F - K.v_b^2)\Delta t = m_b \Delta v_b \quad \& \quad -F \cdot \Delta t = m_r \Delta v_r \quad \text{momentum boat \& rower}$$

$\Rightarrow$

$$m_r \cdot \Delta v_r + m_b \cdot \Delta v_b + K.v_b^2 \cdot \Delta t = 0 \quad \text{momentum} \quad \text{i.e. } \Delta v_b = \frac{m_r \cdot f'(t) - K.v_b^2}{m_r + m_b} \cdot \Delta t$$

Energy & Work considerations

$$\frac{1}{2} m_b v_b^2 + \frac{1}{2} m_r v_r^2 \quad \text{energy before}$$

$$\frac{1}{2} m_b (v_b + \Delta v_b)^2 + \frac{1}{2} m_r (v_r + \Delta v_r)^2 \quad \text{energy after}$$

$$(F - K.v_b^2)v_b \cdot \Delta t + -F \cdot v_r \cdot \Delta t \quad \text{work done}$$

$L$  total sliding distance  
 $T$  time available for sliding  
 $K$  constant fluid dynamic term linking drag force and shell velocity

$v_r$  rower velocity

$v_b$  shell velocity

$\Delta v_b, \Delta v_r$  changes in  $v_b, v_r$  over interval  $\Delta t$

These results have been derived by considering the system, at time  $t$ , as two masses  $m_b$  and  $m_r$ , representing the boat (plus feet) travelling at velocity  $v_b$  and the rower (less feet) travelling at velocity  $v_r$ . Each mass has an equal and opposite force ( $F$ ) acting on it, representing the force exerted by the rower to draw the shell towards him/her. Also, the boat has a water drag force acting on it which has been assumed to be equal to some constant  $K$  times the square of the boat's velocity.  $K$  has been estimated from real data by choosing a value which best fits observed shell deceleration measured using the accelerometers. My data show that  $K$  varies from boat type to boat type.

Consideration of momentum change over a small time interval  $\Delta t$  (during which the velocities have also changed by increments  $\Delta v_b$  and  $\Delta v_r$ ) produces a difference equation which is then integrated with a resolution of  $\Delta t = 0.0001$  seconds to estimate the boat and rower velocities over the entire fixed time interval  $T$  and fixed "sliding distance"  $L$ .  $L$  and  $T$  are held constant for the purposes of these calculations.

Boat and rower velocities derived by integration from:

$$\Delta v_b = \frac{m_r f'(t) - K \cdot v_b^2}{m_r + m_b} \cdot \Delta t \quad \text{and} \quad \Delta v_r = \Delta v_b - f'(t) \Delta t$$

Furthermore, the relative velocity profile of the two masses (aka "sliding speed" or  $(v_b - v_r)$ ), is pre-specified as  $v_b - v_r = f(t)$ , a function of  $t$ . Below are presented the formulae for two of the profiles, both appropriately scaled to cover exactly  $L$  in time  $T$ . The first formula is a sine curve (typical sliding profile), the second a curve of the form  $t^2 \sin(t)$  which effectively shears the whole sine curve to the right aka "late-sliding profile". The "early sliding profile" is created by using the second formula below, but substituting  $\pi - t$  for  $t$  and reversing the sign of the derivative.

$$\text{typical} \quad f(t) = \frac{L \cdot \pi}{2 \cdot T} \cdot \sin\left(\frac{\pi}{T} \cdot t\right) \Rightarrow f'(t) = \frac{L \cdot \pi^2}{2 \cdot T^2} \cdot \cos\left(\frac{\pi}{T} \cdot t\right)$$

$$\text{late sliding} \quad f(t) = \frac{L \cdot \pi}{T^3 \cdot (\pi^2 - 4)} t^2 \cdot \sin\left(\frac{\pi}{T} \cdot t\right) \Rightarrow f'(t) = \frac{L \cdot \pi}{T^3 \cdot (\pi^2 - 4)} \left( 2 \cdot t \cdot \sin\left(\frac{\pi}{T} \cdot t\right) + t^2 \cdot \frac{\pi}{T} \cdot \cos\left(\frac{\pi}{T} \cdot t\right) \right)$$

The key quantity to be determined is the total distance travelled, for different flavours of  $f(t)$ , by the system Centre of Mass ("CoM") during the backstops-to-frontstops interval, starting at a given initial velocity over a given time  $T$  as the boat is drawn back a given relative distance  $L$ . Also calculated are the power consumed by the system as this occurs, and the velocity of the CoM at the end of this interval.

The results should demonstrate if one recovery profile might be different from or better than another.